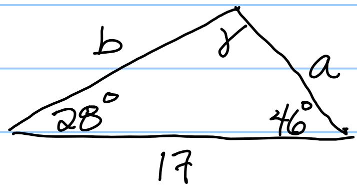
Exam 3 Review Key

Note Title 4/26/2012

1. $\alpha = 28^\circ$, $\beta = 46^\circ$, $c = 17$



$\gamma = 180^\circ - 28^\circ - 46^\circ$ $\boxed{\gamma = 106^\circ}$

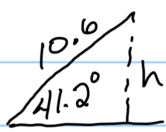
$$\frac{\sin 106^\circ}{17} = \frac{\sin 28^\circ}{a}$$

$$a = \frac{17 \sin 28^\circ}{\sin 106^\circ} \qquad \boxed{a = 8.3}$$

$$\frac{\sin 106^\circ}{17} = \frac{\sin 46^\circ}{b}$$

$$b = \frac{17 \sin 46^\circ}{\sin 106^\circ} \qquad \boxed{b = 12.7}$$

2. $\alpha = 41.2^\circ$, $a = 8.1$, $b = 10.6$



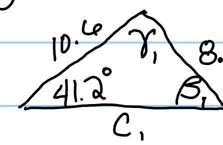
$\sin 41.2^\circ = \frac{h}{10.6}$

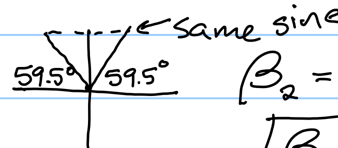
$h = 10.6 \sin 41.2^\circ = 7.0$

Since opposite side is between the height & the adjacent side, there are 2 triangles.

$$\frac{\sin 41.2^\circ}{8.1} = \frac{\sin \beta}{10.6}$$

$$\sin \beta = \frac{10.6 \sin 41.2^\circ}{8.1} = 0.8620$$





$\beta_1 = \sin^{-1}(0.8620)$

$\boxed{\beta_1 = 59.5^\circ}$

$\gamma_1 = 180^\circ - 41.2^\circ - 59.5^\circ$

$\boxed{\gamma_1 = 79.3^\circ}$

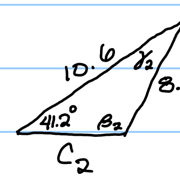
$$\frac{\sin 41.2^\circ}{8.1} = \frac{\sin 79.3^\circ}{c_1}$$

$$c_1 = \frac{8.1 \sin 79.3^\circ}{\sin 41.2^\circ}$$

$\boxed{c_1 = 12.1}$

$\beta_2 = 180^\circ - 59.5^\circ$

$\boxed{\beta_2 = 120.5^\circ}$



$\gamma_2 = 180^\circ - 41.2^\circ - 120.5^\circ$

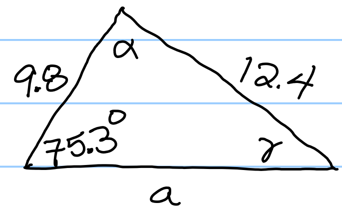
$\boxed{\gamma_2 = 18.3^\circ}$

$$\frac{\sin 41.2^\circ}{8.1} = \frac{\sin 18.3^\circ}{c_2}$$

$$c_2 = \frac{8.1 \sin 18.3^\circ}{\sin 41.2^\circ}$$

$\boxed{c_2 = 3.9}$

3. $\beta = 75.3^\circ$, $b = 12.4$, $c = 9.8$



Since the side opposite the angle is longer than the side adjacent to the angle, there is one triangle.

$$\frac{\sin 75.3^\circ}{12.4} = \frac{\sin \gamma}{9.8}$$

$$\sin \gamma = \frac{9.8 \sin 75.3^\circ}{12.4} = 0.76445$$

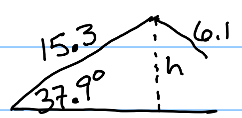
$$\gamma = \sin^{-1}(0.76445) \quad \boxed{\gamma = 49.9^\circ}$$

$$\alpha = 180^\circ - 75.3^\circ - 49.9^\circ \quad \boxed{\alpha = 54.8^\circ}$$

$$\frac{\sin 75.3^\circ}{12.4} = \frac{\sin 54.8^\circ}{a}$$

$$a = \frac{12.4 \sin 54.8^\circ}{\sin 75.3^\circ} \quad \boxed{a = 10.5}$$

4. $\gamma = 37.9^\circ$, $a = 15.3$, $c = 6.1$

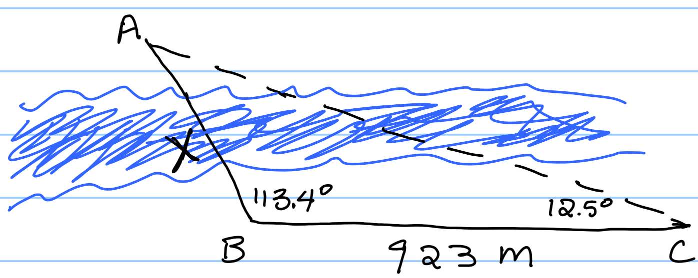


$$\sin 37.9^\circ = \frac{h}{15.3}$$

$$h = 15.3 \sin 37.9^\circ = 9.4$$

Since the side opposite the angle is shorter than the height, there are $\boxed{\text{no triangles}}$

5.



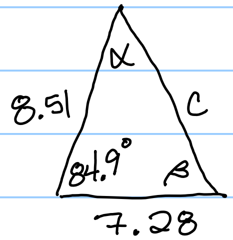
$$\alpha = 180^\circ - 113.4^\circ - 12.5^\circ = 54.1^\circ$$

$$\frac{\sin 54.1^\circ}{923} = \frac{\sin 12.5^\circ}{x}$$

$$x = \frac{923 \sin 12.5^\circ}{\sin 54.1^\circ}$$

$$x = \boxed{246.6 \text{ m}}$$

6.



$$c^2 = 8.51^2 + 7.28^2 - 2(8.51)(7.28)\cos 84.9^\circ$$

$$c^2 = 114.4 \qquad \boxed{c = 10.7}$$

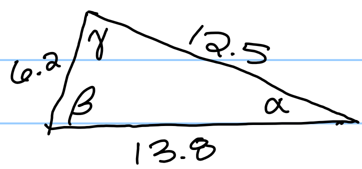
$$\frac{\sin\alpha}{7.28} = \frac{\sin 84.9^\circ}{10.7}$$

$$\sin\alpha = \frac{7.28\sin 84.9^\circ}{10.7} = 0.6779$$

$$\alpha = \sin^{-1}(0.6779) \qquad \boxed{\alpha = 42.7^\circ}$$

$$\beta = 180^\circ - 84.9^\circ - 42.7^\circ \qquad \boxed{\beta = 52.4^\circ}$$

7. $a = 6.2, \quad b = 12.5, \quad c = 13.8$



$$13.8^2 = 6.2^2 + 12.5^2 - 2(6.2)(12.5)\cos\gamma$$

$$190.44 = 194.69 - 155\cos\gamma$$

$$-4.25 = -155\cos\gamma$$

$$\cos\gamma = 0.02742$$

$$\boxed{\gamma = 88.4^\circ}$$

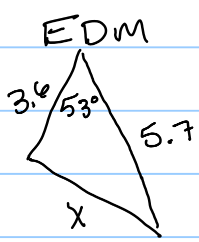
$$\frac{\sin 88.4^\circ}{13.8} = \frac{\sin\alpha}{6.2}$$

$$\sin\alpha = \frac{6.2\sin 88.4^\circ}{13.8} = 0.4491$$

$$\alpha = \boxed{26.7^\circ}$$

$$\beta = 180^\circ - 88.4^\circ - 26.7^\circ \qquad \boxed{\beta = 64.9^\circ}$$

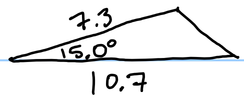
8.



$$x^2 = 3.6^2 + 5.7^2 - 2(3.6)(5.7)\cos 53^\circ$$

$$x^2 = 20.75 \qquad \boxed{x = 4.6}$$

9. $\alpha = 15.0°$ $b = 10.7$ $c = 7.3$



$A = \frac{1}{2}(10.7)(7.3)\sin 15.0°$

$\boxed{A = 10.1 \text{ units}^2}$

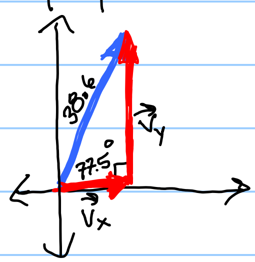
10. $a = 73.5$, $b = 86.4$, $c = 34.9$

$S = \dfrac{73.5 + 86.4 + 34.9}{2} = 97.4$

$A = \sqrt{97.4(97.4 - 73.5)(97.4 - 86.4)(97.4 - 34.9)}$

$\boxed{A = 1265.1 \text{ units}^2}$

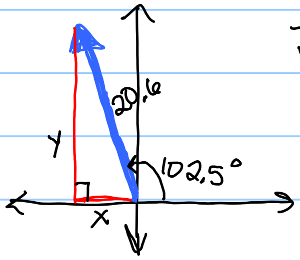
11. $|\vec{v}| = 38.6$ $\theta = 77.5°$



$\cos 77.5° = \dfrac{|\vec{v}_x|}{38.6}$ $|\vec{v}_x| = 38.6\cos 77.5° = \boxed{8.35}$

$\sin 77.5° = \dfrac{|\vec{v}_y|}{38.6}$ $|\vec{v}_y| = 38.6\sin 77.5° = \boxed{37.7}$

12.



$\vec{v} = \langle x, y \rangle$

$\cos 102.5° = \dfrac{x}{20.6}$

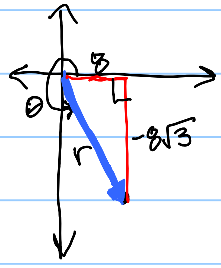
$x = 20.6\cos 102.5° = -4.46$

$\sin 102.5° = \dfrac{y}{20.6}$

$y = 20.6\sin 102.5° = 20.1$

$\boxed{\langle -4.46, 20.1 \rangle}$

13.



$r = \sqrt{8^2 + (-8\sqrt{3})^2}$

$r = \sqrt{64 + 192}$

$r = \sqrt{256}$

$\boxed{r = 16}$

$\sin\theta = \dfrac{-8\sqrt{3}}{16} = -\dfrac{\sqrt{3}}{2}$

$\cos\theta = \dfrac{8}{16} = \dfrac{1}{2}$

$\boxed{\theta = 300°}$

14. $3\langle -1,5\rangle - \langle 4,-7\rangle$

$= \langle -3,15\rangle - \langle 4,-7\rangle$

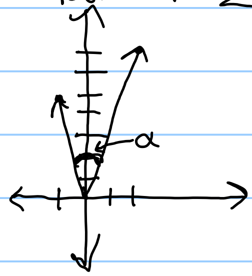
$= \boxed{\langle -7,22\rangle}$

15. $\langle -1,5\rangle \cdot \langle 4,-7\rangle$

$= (-1)(4) + 5(-7)$

$= -4 - 35 = \boxed{-39}$

16. smallest $\angle$ between $\langle -1,5\rangle$ & $\langle 2,7\rangle$



$$\cos\alpha = \frac{\langle -1,5\rangle \cdot \langle 2,7\rangle}{|\langle -1,5\rangle|\,|\langle 2,7\rangle|}$$

$$\langle -1,5\rangle \cdot \langle 2,7\rangle = (-1)(2) + (5)(7) = 33$$

$$|\langle -1,5\rangle| = \sqrt{(-1)^2+5^2} = \sqrt{26}$$

$$|\langle 2,7\rangle| = \sqrt{2^2+7^2} = \sqrt{53}$$

$$\cos\alpha = \frac{33}{\sqrt{26}\sqrt{53}} = 0.88897$$

$$\alpha = \cos^{-1}(0.88897) \quad \boxed{\alpha = 27.3^\circ}$$

17. $\langle 2,-4\rangle$ & $\langle 6,3\rangle$ not parallel (can't multiply one by a constant to get the other)

$\langle 2,-4\rangle \cdot \langle 6,3\rangle = 2(6) + (-4)(3) = 0$ $\boxed{\text{perpendicular}}$

18. $\langle 9,1\rangle$ & $\langle 1,9\rangle$ not parallel

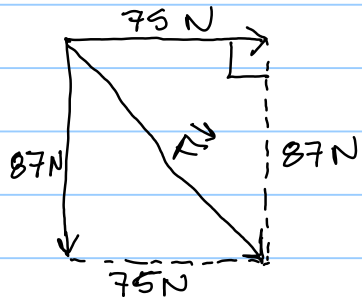
$\langle 9,1\rangle \cdot \langle 1,9\rangle = (9)(1) + (1)(9) = 18$ not perpendicular

$\boxed{\text{neither}}$

19. $\langle -1,7\rangle$ & $\langle 3,-21\rangle$

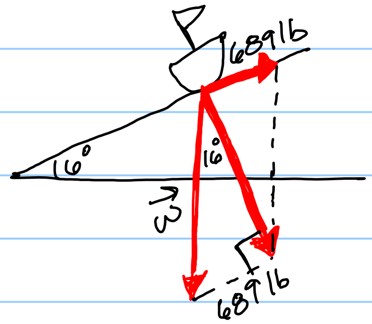
$\boxed{\text{parallel}}$ $-3\langle -1,7\rangle = \langle 3,-21\rangle$

20.



$\vec{F} = \sqrt{75^2+87^2} = \boxed{114.9 \text{ N}}$

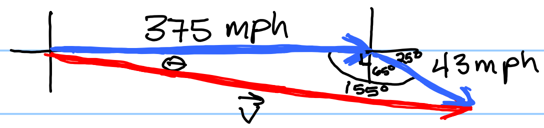
21.

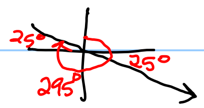


$\sin 16° = \dfrac{689}{|\vec{w}|}$

$|\vec{w}| = \dfrac{689}{\sin 16°} = \boxed{2499.7 \text{ lb}}$

22.





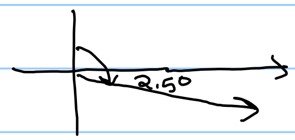
$|\vec{v}|^2 = 375^2 + 43^2 - 2(375)(43)\cos 155°$

$|\vec{v}|^2 = 171{,}702.42$

$|\vec{v}| = \boxed{414.4 \text{ mph}}$

$\dfrac{\sin 155°}{414.4} = \dfrac{\sin \theta}{43}$ $\qquad \sin\theta = \dfrac{43 \sin 155°}{414.4} = 0.0439$

$\theta = 2.5°$



$\text{course} = 90° + 2.5° = \boxed{92.5°}$

23. $(2+7i)-(-4+2i)$

$= 2+7i+4-2i = \boxed{6+5i}$

24. $(4-3i)(2+8i)$

$= 8+32i-6i-\cancel{24}\overset{+24}{}i^2$

$= \boxed{32+26i}$

25. $i^{29} = i^{28} \cdot i = (i^2)^{14} \cdot i = (-1)^{14} \cdot i = \boxed{i}$ OR $29 \div 4 = 7$ R1

26. $i^{76} = (i^2)^{38} = (-1)^{38} = \boxed{1}$ OR $76 \div 4 = 19$ R0

27. $i^{34} = (i^2)^{17} = (-1)^{17} = \boxed{-1}$ OR $34 \div 4 = 8$ R2

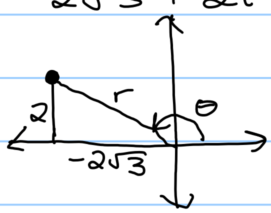
28. $i^{23} = i^{22} \cdot i = (i^2)^{11} \cdot i = (-1)^{11} \cdot i = \boxed{-i}$ OR $23 \div 4 = 5$ R3

29. $2-4i$ conjugate: $2+4i$

$(2-4i)(2+4i) = 2^2+4^2 = \boxed{20}$

30. $\dfrac{12+\sqrt{-36}}{3} = \dfrac{12+6i}{3} = \boxed{4+2i}$

31. $-2\sqrt{3}+2i$



$r = \sqrt{(-2\sqrt{3})^2+2^2}$

$r = \sqrt{12+4} = \sqrt{16}$

$r = 4$

$\cos\theta = \dfrac{-2\sqrt{3}}{4} = -\dfrac{\sqrt{3}}{2}$

$\theta = 150°$

$\boxed{4(\cos 150° + i \sin 150°)}$

32. $9(\cos 240° + i \sin 240°)$

$= 9\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \boxed{-\frac{9}{2} - \frac{9\sqrt{3}}{2}i}$

33. $4(\cos 60° + i \sin 60°) \cdot 3(\cos 150° + i \sin 150°)$

$= 4 \cdot 3\,[\cos(60°+150°) + i \sin(60°+150°)]$

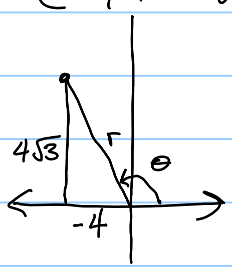
$= 12(\cos 210° + i \sin 210°$

$= 12\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \boxed{-6\sqrt{3} - 6i}$

34. $$\frac{7\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)}{2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)} = \frac{7}{2}\left[\cos\left(\frac{5\pi}{6}-\frac{\pi}{3}\right)+i\sin\left(\frac{5\pi}{6}-\frac{\pi}{3}\right)\right]$$
$$= \frac{7}{2}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$$
$$= \frac{7}{2}(0+i) = \boxed{\frac{7}{2}i}$$

35. $$\left[2(\cos 225^\circ + i\sin 225^\circ)\right]^7 =$$
$$2^7\left[\cos(225^\circ\cdot 7)+i\sin(225^\circ\cdot 7)\right]$$
$$= 128(\cos 1575^\circ + i\sin 1575^\circ$$
$$= 128(\cos 135^\circ + i\sin 135^\circ$$
$$= 128\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$$
$$= \boxed{-64\sqrt{2}+64i\sqrt{2}}$$

36. $(-4+4i\sqrt{3})^4$



$$r=\sqrt{(4\sqrt{3})^2+(-4)^2}$$
$$r=\sqrt{48+16}=\sqrt{64}$$
$$r=8$$
$$\cos\theta=\frac{-4}{8}=-\frac{1}{2}$$
$$\theta=120^\circ$$

$$\left[8(\cos 120^\circ + i\sin 120^\circ)\right]^4$$
$$= 8^4\left[\cos(4\cdot 120^\circ)+i\sin(4\cdot 120^\circ)\right]$$
$$= 4096(\cos 480^\circ + i\sin 480^\circ)$$
$$= 4096(\cos 120^\circ + i\sin 120^\circ)$$
$$= 4096\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$
$$= \boxed{-2048+2048i\sqrt{3}}$$

37. Fourth roots of $81(\cos 292^\circ + i\sin 292^\circ)$

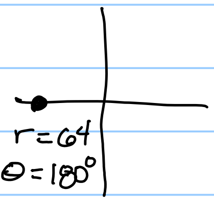
$81^{1/4}=3$ $\qquad \frac{292^\circ+360^\circ k}{4} = 73^\circ + 90^\circ k$

$$\boxed{3(\cos\alpha + i\sin\alpha) \text{ for } \alpha = 73^\circ, 163^\circ, 253^\circ}$$

38. $x^3 + 64 = 0$

$x^3 = -64$

cube roots of -64



$64(\cos 180° + i \sin 180°)$

$64^{1/3} = 4$

$\frac{180° + 360° k}{3} = 60° + 120° k$

$60°, 180°, 300°$

$4(\cos 60° + i \sin 60°) = 4\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) = \boxed{2 + 2i\sqrt{3}}$

$4(\cos 180° + i \sin 180°) = 4(-1 + 0i) = \boxed{-4}$

$4(\cos 300° + i \sin 300°) = 4\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = \boxed{2 - 2i\sqrt{3}}$